

## Damage spreading for one-dimensional, nonequilibrium models with parity conserving phase transitions

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The damage spreading (DS) transitions of two one-dimensional stochastic cellular automata suggested by Grassberger (*A* and *B*) and the nonequilibrium kinetic Ising model of Menyhárd (NEKIM) have been investigated. These nonequilibrium models exhibit nondirected percolation universality class continuous phase transitions to absorbing states, exhibit parity conservation (PC) law of kinks, and have chaotic to nonchaotic DS phase transitions, too. The relation between the critical point and the damage spreading point has been explored with numerical simulations. For model *B* the two transition points are well separated and directed percolation universality was found both for spin damage and kink damages in spite of the conservation of damage variables modulo 2 in the latter case. For model *A* and NEKIM the two transition points coincide with drastic effects on the damage of spin and kink variables showing different time dependent behaviors. While the kink DS transition of these two models shows regular PC class universality, their spin damage exhibits a discontinuous phase transition with compact clusters and PC-like spreading exponents. In the latter case the static exponents determined by finite size scaling are consistent with that of the spins of the NEKIM model at the PC transition point. The generalized hyperscaling law is satisfied. Detailed discussion is given concerning the dependence of DS on initial conditions especially for the *A* model case, where extremely long relaxation time was found. [S1063-651X(98)12605-3]

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### I. INTRODUCTION

While damage spreading (DS) was first introduced in biology [1] it has become an interesting topic in physics as well [2–4]. The main question is if a damage introduced in a dynamical system survives or disappears. To investigate this the usual technique is to make replica(s) of the original system and let them evolve with the same dynamics and external noise. This method has been found to be very useful to measure accurately dynamical exponents of equilibrium systems [5]. It has turned out, however, that the DS properties do depend on the applied dynamics. An example is the case of the two-dimensional Ising model with heat-bath algorithm versus Glauber dynamics [6–8].

To avoid the dependences on dynamics Hinrichsen *et al.* [9] suggested a definition of “physical” family of DS dynamics according to which the active phase may be divided into a subphase within which DS occurs for every member of the family, another subphase where the damage heals for every member of the family, and a third possible subphase where DS is possible for some members and the damage disappears for other members. The family of possible DS dynamics is defined such that it is to be consistent with the physics of the single replicas (symmetries, interaction ranges, etc.).

The universality of continuous DS transitions is an other open question. There is a hypothesis raised by Grassberger [10], that damage spreading transitions generically belong to the universality class of directed percolation (DP) if they are separated from the ordinary critical point. This claim is based on the DP hypothesis applied for the absorbing type

DS transition since we can consider the difference of the replicas as another dynamical system evolving by a complex rule. According to the DP hypothesis—conjectured first in the early 1980s [11,12]—in the absence of conservation laws every continuous phase transition of a system with scalar order parameter and local interactions to a single absorbing phase would belong the universality class of the DP. There are other more complex models such as those with several absorbing states [13] and multicomponent systems [14], which exhibit the DP transition too. The DP universality class has been proven to be extremely robust. For a long time only a few number of exceptions has been found, which do not belong to the DP class. These are the parity conservation (PC) models and the multiplicative noise systems [15].

The first examples of the PC models were Grassberger’s (*A* and *B*) stochastic cellular automata (SCA) [16]. The kinks (00’s and 11’s) of these models exhibit mod 2 parity conservation and the absorbing state is doubly degenerated. Following that a series of models in the same universality class have been discovered. In the case of the branching annihilating random walk with an even number of offspring (BAWe) [17,18] the parity of the “particles” is conserved and there is a single absorbing state. In the nonequilibrium Ising model with combined spin-flip and spin exchange dynamics (NEKIM) [19,20] the kinks have local parity conserving symmetry and the absorbing state is symmetrically doubly degenerated. The three species monomer-monomer [21] (3MM) and the interacting monomer-dimer (IMD) models [23,24] are multicomponent models with parity conservations and symmetric absorbing phases.

The common feature of these models that force them in a

TABLE I. Summary of PC related models. The notation  $k$  refers to kink,  $s$  refers to spin, and  $h$  means external field as variable of the model.

Model	Abs. state	Dynamics	Univ.	Ref. no.
A-k	$Z_2$ symm.	BAWe	PC	[16]
B-k	$Z_2$ symm.	BAWe	PC	[16]
BAWe-s	singlet	BAWe	PC	[17,18,25,26]
BAWo-s	singlet	BAWo	DP	[17,18,25,26]
NEKIM-k	$Z_2$ symm.	BAWe	PC	[19,27,38]
3MM-s	$Z_2$ symm.	BAWe	PC	[21]
IMD-s	$Z_2$ symm.	BAWe	PC	[23,24]
GDK-s	$Z_2$ symm.	BAWe	PC	[31]
ISCA-s	singlet	Global-PC	DP	[22]
IMD-s+h	$Z_2$ broken	BAWe	DP	[24]
NEKIM-k+h	$Z_2$ broken	BAWe	DP	[27]
GDK-s+h	$Z_2$ broken	BAWe	DP	[31]

non-DP universality class was first conjectured to be the parity conservation (the PC class name comes from here). However, this had to be refined, because models were found with global parity conservation but DP class phase transition [see, for example, Ref. [22] (ISCA)]. Field theoretical investigations of the BAW models showed that the BAWe parity conservation dynamics in one dimension results in a new non-DP fixed point possessing the PC class universality, while for an odd number of offspring (BAWo) the transition is in the DP class [25,26]. Furthermore, among the multiple absorbing state models one can have DP behavior and BAWe parity conservation dynamics together if the symmetry of the absorbing states is broken [see [24] in the case of IMD, [27] in case of the NEKIM model, and [31] in case of the generalized Domany-Kinzel SCA (GDK)]. This implies that for multiabsorbing state models the BAWe parity conservation is not a sufficient condition to have non-DP universality class but the symmetry of the ground state is necessary too. See Table I.

A very recent study has shown that the DS transition is possible in a one-dimensional non-equilibrium kinetic Ising model [32] too, and the universality class of the transition is not always in the DP class. The dynamics was engineered as the combination of two subrules such that it creates  $Z_2$  symmetric passive states, the kink damage variables follow BAWe parity preserving dynamics, and a PC universality class DS transition emerges.

In this work we have investigated the damage spreading behavior of some one-dimensional PC models: Grassberger's  $A$  and  $B$  stochastic cellular automata (SCA) [5] with synchronous dynamics and the NEKIM model [19]. The "kink" variables of these models possess parity conservation and continuous PC class phase transition.

In the case of the NEKIM model the 01 and 10 pairs act as kinks and follow the basic (BAWe) elementary reactions: (i) left-right diffusion, (ii)  $X \rightarrow 3X$  reproduction, (iii)  $2X \rightarrow 0$  annihilation.

Since at damage spreading problems we follow the evolution of two or more replicas, we can consider it as a special, multicomponent dynamical problem (here with multiple absorbing states too). Furthermore, when the DS point is inside the active phase there is a passive state of the damage

variables with fluctuating replicas in the background. Therefore the simple DP universality hypothesis cannot be applied here (although this does not exclude a DP class phase transition).

We also studied here the DS properties of spins of these systems, which can be regarded as the "dual" variables of kinks but they do not obey the parity conservation. The combined observation of spin and kink damage variables sheds light on the interplay of parity conservation, absorbing state symmetry and universality. In this paper we shall show by numerical simulations that the universality is determined not only by the dynamics but that the symmetry of the absorbing state is a necessary condition again as in [24,27].

If the critical point and the DS point coincide interesting things happen. While the kink damage exponents will belong to the PC universality class, in the case of the spin damage the static exponents determined by finite size scaling are in agreement with that of the pure NEKIM model at the PC transition point on the spin level [27]. Detailed discussion on this is forthcoming [33].

The dependence of the DS results on the initial states of the replicas is discussed because for  $A$  model very slow relaxation makes it an important point.

## II. DAMAGE SPREADING SIMULATION METHODS

The time dependent simulation is a well established method to locate critical points and to measure dynamical critical exponents at the same time [28]. Here we applied it for kink and spin damage variables for system sizes  $L = 4096 - 16384$  with periodic boundary conditions. A single spin-flip difference is introduced between two identical replicas at the beginning of each simulation runs. The difference of spin and kink variables is measured during a time evolution with identical rules and random numbers for both replica. The maximum number of simulation steps was chosen to be  $t_{\text{MAX}} = L/2$ , and so the damage variables cannot reach the boundaries and one can avoid finite size effects of them. However, simulating near the critical point causes long transients, hindering one from seeing the true scaling behavior within reachable times.

The role of initial states of the replicas is not discussed in the DS simulation literature. If the DS transition point is not in the neighborhood of a critical point an exponentially quick transient to the steady state is expected, but if they coincide—as in case of the Grassberger  $A$  model—the evolution to steady state slows down to power law time dependence and we can expect finite time effects. First random states have been chosen with equal and uniform distribution of 0's and 1's. In the case of model  $A$  SCA this resulted in very confusing results. Then we investigated the effects by starting with a steady state configurations, i.e., replicas were driven to steady state before the DS measurements.

The quantities characterizing damage evolution show powerlike behavior in the  $t \rightarrow \infty$  limit at the damage spreading point ( $p_d$ ) separating chaotic and nonchaotic phases. The Hamming distance will be the order parameter of this paper:

$$D(t) = \left\langle \frac{1}{L} \sum_{i=1}^L |s(i) - s'(i)| \right\rangle, \quad (1)$$

where  $s(i)$  may denote now spin or kink variables. Kink variables for these models are the 00 and 11 pairs in the case of the  $A, B$  SCA and 01 and 10 pairs for the NEKIM model. If there is a phase transition point, the Hamming distance behaves in a power law manner at that point:

$$D(t) \propto t^\eta. \quad (2)$$

Similarly the survival probability of damage variables behaves as

$$P_s(t) \propto t^{-\delta} \quad (3)$$

and the average mean square distance of damage spreading from the center scales as

$$R^2(t) \propto t^z. \quad (4)$$

The evolution runs were averaged over  $N_s$  independent runs for each different value of  $p$  in the vicinity of  $p_d$  [but for  $R^2(t)$  only over the surviving runs].

To estimate the critical exponents and the transition points together we determined the local slopes of the scaling variables. For example, in case of the survival probability,

$$-\delta_p(t) = \frac{\ln[P_s(t)/P_s(t/m)]}{\ln(m)} \quad (5)$$

and we have used  $m=4$ . In the case of power-law behavior we should see a horizontal straight line as  $1/t \rightarrow 0$ , when  $p = p_d$ . The off-critical curves should possess curvature. Curves corresponding to  $p > p_d$  should veer upward, curves with  $p < p_d$  should veer downward.

The damage spreading measurement of the order parameter time scaling can be very effectively parallelised in a multispin code manner [10], since one needs only one random number for each site of the different replicas and so one can follow the evolution of  $N_r = (32 \times 31)/2$  replicas in a simple 32-bit computer vector of length  $L$ . However, this method is not applicable to measure the survival probability scaling and  $z$ , since the healing of differences among all the  $N_r$  replicas takes a very long time and one cannot introduce a single initial damage for each pair at the center of the lattice. For the simulation of survival probability a very effective code has been implemented for a special, associative string processor [34].

To determine static exponents finite-size scaling (FSS) simulations were performed as well. As shown by Aukrust *et al.* [35,36], FSS is applicable to continuous, nonequilibrium phase transitions. At the critical point the order parameter steady state density ( $D$ ) and the fluctuation  $\chi = L^d(\langle D^2 \rangle - \langle D \rangle^2)$  scale with the system size as

$$D(L) \propto L^{-\beta/\nu_\perp}, \quad (6)$$

$$\chi(L) \propto L^{\gamma/\nu_\perp}, \quad (7)$$

where  $\nu_\perp$  is the correlation length exponent in the space direction:

$$\xi(p) \propto |p - p_c|^{-\nu_\perp}, \quad (8)$$

$\beta$  is the order parameter exponent in the steady state:

$$D(p) \propto |p - p_c|^\beta, \quad (9)$$

and  $\gamma$  describes the fluctuation of it:

$$\chi(p) \propto |p - p_c|^{-\gamma}. \quad (10)$$

Simulations were done in one dimension for lattice sizes  $L = 64, 128, \dots, 2048$ . The necessary time steps to reach steady state were determined experimentally. The time evolution of the concentration was plotted, and the necessary time steps were fixed for a given  $L$  such as 200–500 time values following the level off. Averaging was done for these 200–500 values times the number of surviving samples (500). The  $p_d$  values were taken from the time-dependent Monte Carlo (MC) calculations.

The dynamic exponent  $Z = \nu_\parallel / \nu_\perp$  can be determined from the FSS of the characteristic time  $\tau(p, L)$ . In this study we measured the time necessary to reach half of the steady state concentration starting from a single damage state. The characteristic time obeys the finite size scaling law:

$$\tau(p, L) \propto L^Z h[(p - p_c)L^{1/\nu_\perp}], \quad (11)$$

where  $Z = \nu_\parallel / \nu_\perp$ . For this measurement we used the same damage concentration time evolutions as in the case of the static runs above.

### III. GRASSBERGER B MODEL

A (BAWe) parity conserving dynamics can be realized on the kinks (or ‘‘particles’’) of the following SCA (we show the configurations at  $t-1$  and the probability of getting 1 at time  $t$ ):

$$\begin{array}{rcccccccc} t-1: & 100 & 001 & 101 & 110 & 011 & 111 & 000 & 010 \\ t: & 1 & 1 & 1 & p & p & 0 & 0 & 0 \end{array}$$

The 00 and 11 pairs are the simplest kinks of the model. The time evolution pattern for  $p=0$  is a regular chessboard in  $1+1$  dimensions (Rule-50 with double degeneration), i.e., the absorbing states are period-two antiferromagnetic. For  $p < p_c [= 0.539(1)]$  kinks disappear exponentially, while for  $p > p_c$  they survive with a finite concentration. In the  $p=1$  limit we get the deterministic Rule-122, which is known to be chaotic. So there is damage spreading phase transition besides the absorbing phase transition of PC universality.

#### A. Kink damage results

First the simulations were started from two replicas of lattices with identical random initial states but with a 2-kink initial difference. The parity of the lattice forces an even or odd number of initial kinks, therefore it is not possible to create odd numbered kink differences. The parity of kinks is conserved. The parity of kink differences (even) is conserved too.

Still we see a DP-like universality of the damage variables (Figs. 1 and 2). The location of the damage spreading point [ $p_d = 0.632(1)$ ] is far from the PC critical point [ $p_c = 0.539(1)$ ], therefore the active phase is divided into a chaotic and nonchaotic subphase similarly to the case of the Domany-Kinzel SCA [29,30,10]. The replicas at the DS

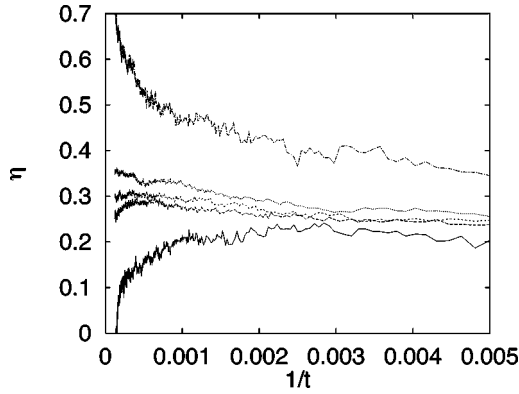


FIG. 1. Local slopes of the Hamming distance ( $\eta$ ) in the  $B$  model, for  $p=0.62, 0.63, 0.632, 0.634, 0.65$  (curves from bottom to top). Statistical averaging was done over 10 000 samples.

point are in fluctuating states, therefore, they do not have the “chessboard” double degeneration as in case of the PC critical point. The emergence of the DP exponents in spite of the mod 2 conservation of kink damage variables is similar to what was found numerically and analytically for PC models, when the external  $H$  field destroyed the symmetry of the absorbing state [24,27,31]. This suggests that the BAWe parity conservation rule is not a sufficient condition for having non-DP universality.

The same results have been found if we started the replicas from steady states with a single spin-flip initial difference.

### B. Spin damage

Following the damage (difference) of the spins, instead of the kinks, we obtained the same DP-like results. The universality was insensitive to the parity of the damage variables, it is DP for both cases.

### C. Finite size scaling for both cases

Finite size scaling simulations were performed at the DS transition point ( $p_d=0.633$ ) for system sizes  $L=64, 128, \dots, 1024$ . The necessary time steps to reach steady state were  $t=40\,000, 80\,000, \dots$ , respectively. The results can be seen in Fig. 3.

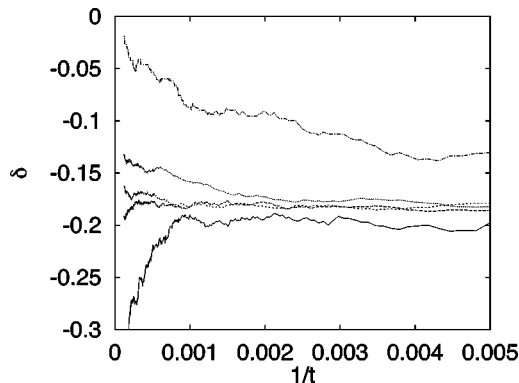


FIG. 2. Local slopes of the damage survival probability ( $\delta$ ) in the  $B$  model, for  $p=0.62, 0.63, 0.632, 0.634, 0.65$  (curves from bottom to top). Statistical averaging was done over 10 000 samples.

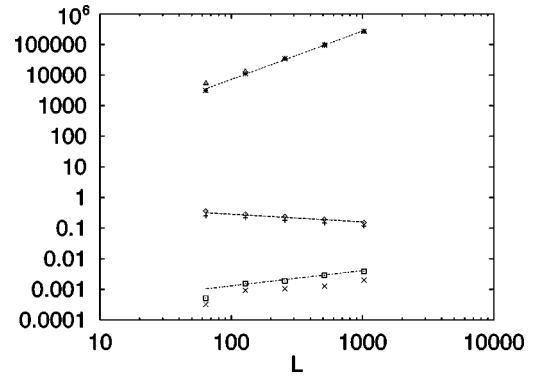


FIG. 3. Finite size scaling results for the spin and kink damage for the  $B$  model. The diamonds correspond to the kink damage concentration, the squares to the fluctuations of it. The crosses correspond to the spin damage concentration, the  $x$ 's to the fluctuations of it. Triangles and stars denote characteristic times  $\tau$  of the spin and the kink damage cases. Averaging was done over 500 surviving samples.

As one can see both the kink and spin damage concentration show a scaling with  $-\beta/\nu_{\perp} = -0.25$ , while the fluctuations have a slope  $\gamma/\nu_{\perp} = 0.5$ , all agreeing with the corresponding exponents [37] in the DP universality class. For the critical dynamical exponent  $Z = \nu_{\parallel}/\nu_{\perp}$  DP-like scaling has been found for both the spin and kink damage cases. Fitting can be done with  $L^{1.5798}$  (DP value) in both cases.

## IV. GRASSBERGER A MODEL

Another very similar model exhibiting parity conservation of kinks is the Grassberger A stochastic cellular automaton:

$$\begin{array}{l} t-1: \quad 100 \quad 001 \quad 101 \quad 110 \quad 011 \quad 111 \quad 000 \quad 010 \\ t: \quad \quad 1 \quad 1 \quad 0 \quad 1-p \quad 1-p \quad 0 \quad 0 \quad 1 \end{array}$$

The time evolution pattern in 1+1 dimensions, for small  $p$  evolves towards a stripelike ordered steady state (with double degeneration), while for  $p > p_c [=0.1245(5)]$  the kinks (the 00 and 11 pairs) survive. For  $p=0$  we have the Rule-94, class 1 CA, while the  $p=1$  limit is the chaotic Rule-22 deterministic CA. Therefore we can expect a dam-

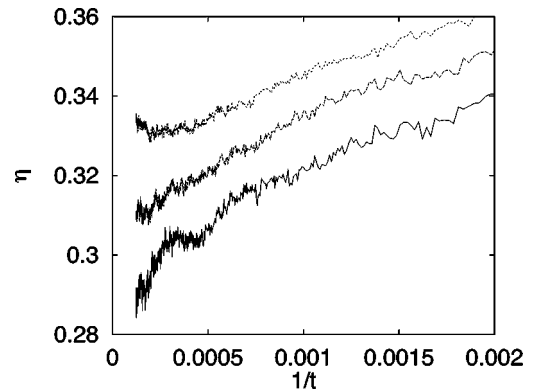


FIG. 4. Local slopes of the Hamming distance ( $\eta$ ) in the  $A$  model, for  $p=0.130, 0.132, 0.134$  (curves from bottom to top). The simulations were started from the random initial state. Statistical averaging was done over  $6 \times 10^5$  samples.

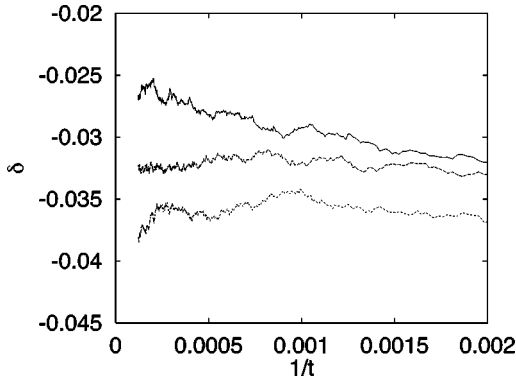


FIG. 5. Local slopes of the damage survival probability ( $\delta$ ) in the A model, for  $p=0.130, 0.132, 0.134$  (curves from bottom to top). The simulations were started from the random initial state. Statistical averaging was done over  $10^4$  samples.

age spreading phase transition between  $p=p_c$  and  $p=1$  (damages always heal or survive in the ordered steady states).

#### A. Kink damage results

First two replicas of lattices of the same random initial distributions but a single spin-flip difference condition were followed. As the local slope figure (Fig. 4) of the  $D(t)$  shows the DS transition point [ $p_d=0.133(1)$ ] seems to be slightly off the critical point [ $p_c=0.1242(5)$ ]. One can read off  $\eta=0.31(1)$ , which is close to the DP universality class value [ $\eta_{DP}=0.314(3)$ ], but for the survival probability we got nearly zero exponent (Fig. 5). The survival probability scaling with  $\delta\sim 0$  contradicts the DP scaling and one may speculate that we can see a finite time effect. We extended the same time dependent simulations up to  $t_{\max}=28\,000$  for certain  $p$  values, but there were no sign of change in the above results. To check the transients the evolution of the kink concentration starting from a disordered state has been followed on a  $L=8192$  lattice. As Fig. 6 shows there is a very long relaxation in this model, and the steady state has been reached following  $2\times 10^6$  MC time steps only. Therefore time dependent simulations from steady state initial conditions have been performed. The initial states now were chosen to be the outcomes of runs following  $5\times 10^6$  time steps for different  $p$  s, with the usual single spin-flip difference.

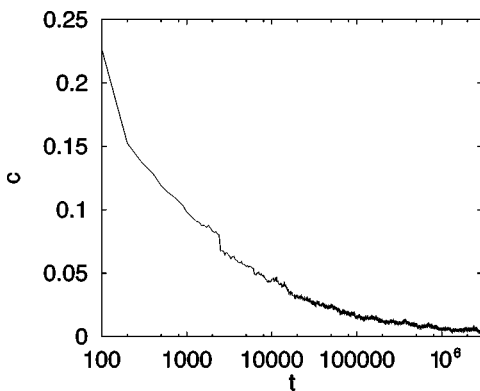


FIG. 6. Evolution of the kink concentration in the A model ( $L=8192$ ) started from random initial state at  $p=0.124$ .

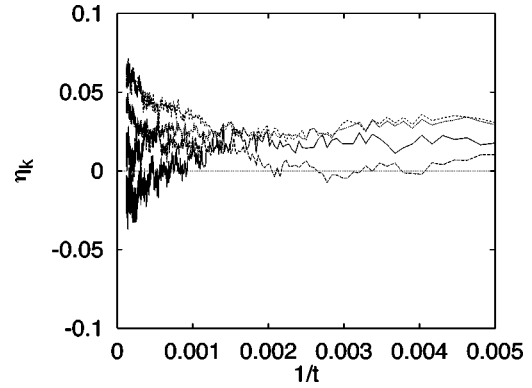


FIG. 7. Local slopes of the Hamming distance ( $\eta_{\text{kink}}$ ) in the A model, for  $p=0.123, 0.124, 0.125, 0.126$  (curves from bottom to top) in the case of steady state initial condition. Averaging was done over  $3\times 10^5$  independent samples.

Now we can see dramatic changes. First, the DS point moves to the critical point [ $p_d=0.1242(1)$ ] (see Fig. 7).

The corresponding  $\eta_k$  exponent is around zero, which agrees with that of the PC universality class. In case of the survival probability we could use the conventional non-multi-spin coding algorithm with much less statistics. Still one can read off the same transition point with the value  $\delta\sim 0.285(8)$  (Fig. 8), which is again in the PC class.

Thus we can see the emergence of PC behavior, which is in accordance with the BAWe conservation of kink-damage variables and the  $Z_2$  degeneration of the absorbing state arises from the fact that  $p_c$  and  $p_d$  coincided. Note that the statistical errors are larger now than in the B model case, when the DS simulations were carried out not in the immediate neighborhood of the critical point.

#### B. Spin damage results

The parity of the spin damage variables is not conserved in this case. When the simulations were started from random initial states we obtained the same DS transition point as in case of the kink-damage case, but with neither DP nor PC universality class values. The simulations have been done both with conventional and multispin code algorithm. These resulted in the results for the spin damage Hamming distance

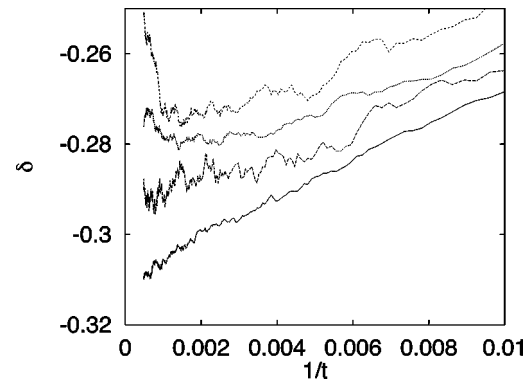


FIG. 8. Local slopes of the survival probability ( $\delta$ ) in the A model, for  $p=0.123, 0.124, 0.1245, 0.125$  (curves from bottom to top) in case of steady state initial condition. Averaging was done over 50 000 independent samples.

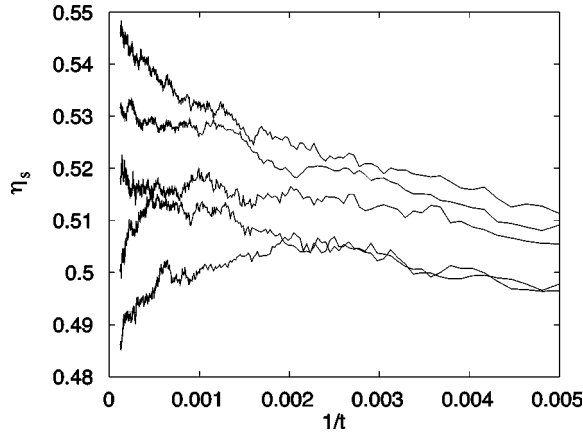


FIG. 9. Local slopes of the Hamming distance ( $\eta$ ) in the A model, for  $p=0, 125, 0, 127, 0.130, 0.132, 0.134$  (curves from bottom to top). The simulations were started from the random state. Statistics over 100 000–500 000 samples.

shown in Fig. 9. For the survival probability we obtained a nearly zero  $\delta$  exponent, as in the case of kink damage; see Fig. 10. These results are quite confusing again, especially the exponent  $\eta=0.52(1)$ , which does not belong to the  $(1+1)$ -dimensional DP or to the PC class. We cannot give a better explanation for this, that the very long transients prevented the healing of damages and the possibility to see the “true” scaling behavior.

Indeed, if the simulations were started from near the steady state the results become very different. The DS transition point seems to coincide with the critical point [ $p_c=0.1242(5)$ ] and we could get a spin-damage concentration exponent:  $\eta_s=0.29(2)$ , which is close to the  $\eta'=0.285(5)$  of the PC scaling (Fig. 11).

The results for the survival probability ( $\delta_s$ ) and  $z_s$  coincided with that of the kink-damage case, which can be understood by the following. Although theoretically to each spin-damage absorbing state can correspond two kink-damage absorbing states (by flipping all spins of one replica), simulations showed that the kink and the spin damage died out always at the same time. In the case of the spreading one can easily check that the  $R^2$  measurements should give the same results for both kink and spin damage cases, because the beginning and the end of the perturbed region are

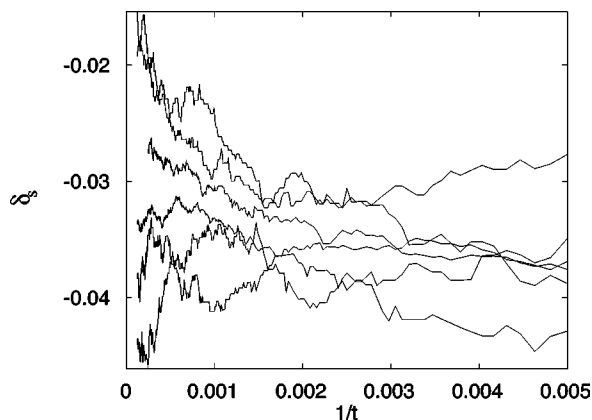


FIG. 10. Same as above for the survival probability.

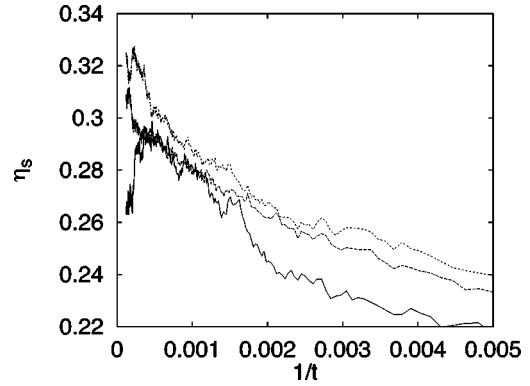


FIG. 11. Local slopes of the Hamming distance ( $\eta_s$ ) in the A model, for  $p=0, 124, 0, 1245, 0.126$  (curves from bottom to top). The simulations were started from steady state. Averaging was over  $3 \times 10^5$  samples.

almost the same. Indeed the simulations resulted in the same PC-like  $z$  [ $z=1.14(1)$ ] exponent in both cases (Fig. 12).

### C. Finite size scaling for both cases

The finite size scaling simulations were performed at the DS transition point ( $p_d=0.1242$ ) for system sizes  $L=64, 128, \dots, 1024$ . The necessary time steps to reach steady state were  $t=10\,000, 20\,000, \dots$  respectively. The results can be seen in Fig. 13.

In the case of the kink damage one can see regular PC-like scaling for the concentration  $-\beta_k/v_\perp=-0.5$ , the fluctuations of it  $\gamma_k/v_\perp=0$ , and for the critical dynamical exponent  $Z_k=v_\parallel/v_\perp=1.75$ . In the case of the spin damage we can see a constant 0.5 steady state concentration for all system sizes, resulting in  $\beta_s/v_\perp=0$  as in the case of the pure Glauber Ising model at  $T=0$  and the NEKIM model at the PC transition point. In agreement with this and Fisher’s static scaling law

$$\gamma=dv_\perp-2\beta \quad (12)$$

the fluctuations of it exhibit a linear scaling law ( $\gamma_s/v_\perp=1$ ), whereas the scaling of the characteristic time is described by what was found in the case of the pure NEKIM model at the PC transition point for the spin variables [27].

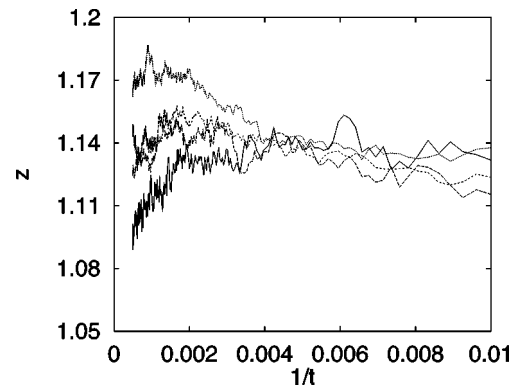


FIG. 12. Local slopes of  $R^2(t)$  ( $z$ ) in the A model, for  $p=0, 124, 0, 1245, 0.125, 0.126$  (curves from bottom to top). The simulations were started from steady state. Averaging was done over  $5 \times 10^4$  independent run.

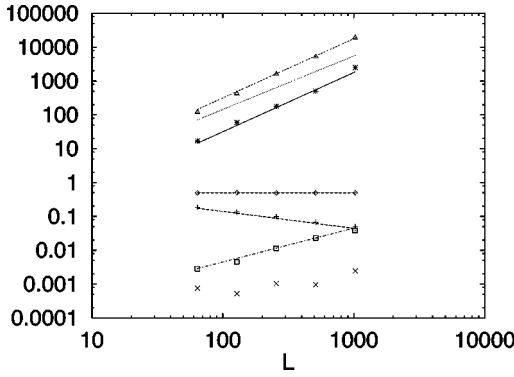


FIG. 13. Finite size scaling results for the spin and kink damage for the A model. The diamonds correspond to the spin damage concentration, the squares to the fluctuations of it. The crosses correspond to the kink damage concentration, the  $\times$ 's to the fluctuations of it. Triangles and stars denote characteristic times  $\tau$  of the spin and the kink damage cases. Averaging was done over 500 surviving samples.

### V. NEKIM MODEL

The PC universality appears in a class of nonequilibrium dynamic Ising models where the kinks corresponding to 01 and 10 domain walls evolve according to the BAWe rules [19]. The dynamics is composed of the alternating application of (i) a zero temperature spin flip lattice update:

$$w_i = \frac{\Gamma}{2} (1 + \delta s_{i-1} s_{i+1}) \left( 1 - \frac{\gamma}{2} s_i (s_{i-1} + s_{i+1}) \right), \quad (13)$$

where  $\gamma = \tanh 2J/kT$  ( $J$  denoting the coupling constant in the Ising Hamiltonian),  $\Gamma$  and  $\delta$  are further parameters resulting in random walk, annihilation of kinks; (ii) and a spin-exchange lattice update:

$$w_{ii+1} = \frac{1}{2} p_{\text{ex}} [1 - s_i s_{i+1}], \quad (14)$$

where  $p_{\text{ex}}$  is the probability of spin exchange, resulting in kink  $\rightarrow$  3 kink creation.

The spin-flip part has been applied using two-sublattice updating, while  $L$  MC spin-exchange attempts has been done randomly using the outcome state of the spin-flip part. All these together have been counted as one time step of exchange updating. [Usual MC update in this last step en-

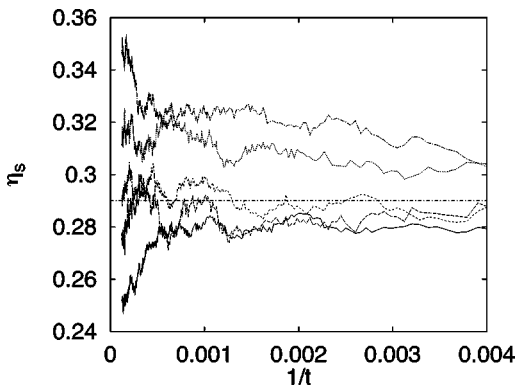


FIG. 14. Local slopes of the spin damage concentration ( $\eta_s$ ), for  $p=0.386, 0.39, 0.392, 0.395, 0.4$  (curves from bottom to top). Statistical averaging was done over 20 000–40 000 samples.

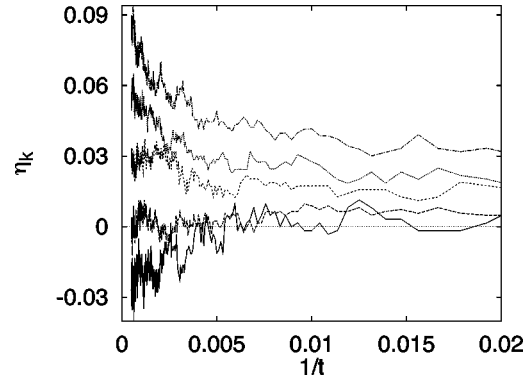


FIG. 15. Local slopes of the kink damage concentration ( $\eta_k$ ), for  $p=0.385, 0.39, 0.395, 0.405$  (curves from bottom to top). Statistical averaging was done over 100 000 samples.

hances the effect of  $p_{\text{ex}}$  and leads to  $\delta_c = -0.362(1)$ .] In [19,27] we have started from a random initial state and determined the phase boundary in the  $(\delta, p_{\text{ex}})$  plane. The phase space is composed of an active phase with free kinks and an absorbing, ordered phase without kinks provided the initial state has an even number of kinks. They are separated by a second order phase transition line of PC universality. To investigate the damage spreading properties we have chosen to fix  $\Gamma=0.35$ ,  $p_{\text{ex}}=0.3$ , and change  $\delta$  (that will play the role of  $p$  now). The PC critical point has been determined precisely [38]:  $\delta_c = -0.395(2)$ .

### A. Time dependent simulations

Time dependent simulations up to  $t_{\text{MAX}}=8192$  were performed and we found that the DS transition point coincides with  $\delta_c$  within statistical errors. The simulation results now were less sensitive whether we started from random initial state or from steady state. For the spin damage density we obtained  $\eta_s=0.29(1)$  when we started from steady state, as in the case of the A model. [If we started from the random initial state  $\eta_s=0.38(2)$  scaling appeared at the  $\delta_c$  point.] This is shown in Fig. 14.

The Hamming distance measurements for kinks resulted in  $\eta \sim 0$  in accordance with the PC universality class value (Fig. 15). The survival probability simulation gave the same

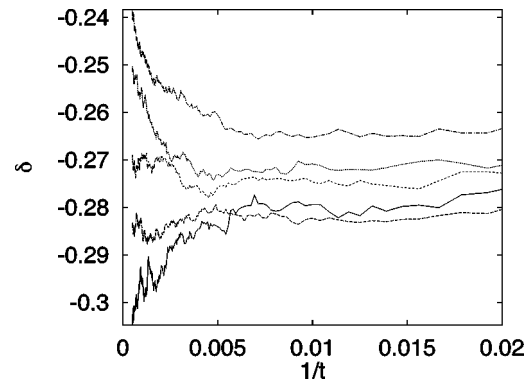


FIG. 16. Local slopes of the damage spin and kink survival probability ( $\delta$ ), for  $p=0.385, 0.39, 0.395, 0.405$  (curves from bottom to top). Statistical averaging was done over 100 000 samples.

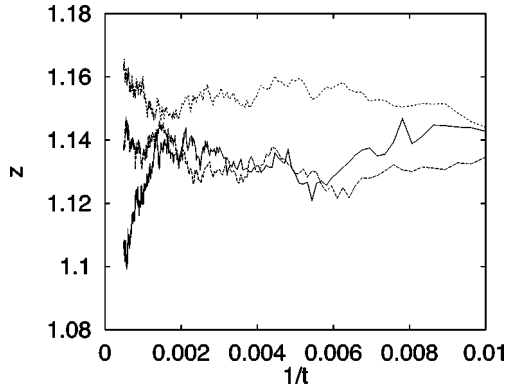


FIG. 17. Local slopes of  $R^2$  of spin damage ( $z$ ), for  $p=0.39, 0.395, 0.4$  (curves from bottom to top). Statistical averaging was done over 100 000 samples.

results for spin and kink damage cases, namely, they are PC-like; see (Fig. 16). As in case of the  $A$  model the simulations resulted in the same PC-like  $z$  [ $z=1.14(1)$ ] (see Fig. 17) exponent in both cases, because the spin and kink damage regions start and stop approximately at the same sites. This can be observed on Figs. 18 and 19, where we plotted the time evolution patterns of spin and kink damages of the same run.

One can see that the boundaries of the perturbed regions are the same, but the spin-damage pattern is compact, causing the higher  $\eta'_s$  exponent.

### B. Finite size scaling

Finite size scaling simulations were performed at the DS transition point ( $p_d=0.395$ ) for system sizes  $L=64, 128, \dots, 2048$ . The necessary time steps to reach steady state were  $t=7000, 14\,000, \dots, 200\,000$ , respectively. Figure 20 summarizes the results. The kink concentration shows a nice scaling with  $\beta_k/\nu_\perp=0.5$ , while the fluctuations had no pronounced slope on the log-log plot, suggesting  $\gamma_k/\nu_\perp=0$ . Both of these values are in agreement with the PC universality class. The spin damage concentration is constant [ $c=0.50(1)$ ] meaning  $\beta_s/\nu_\perp=0.0$  similarly to the  $A$ -model case. A linear scaling with  $\gamma_s/\nu_\perp=1$  could be fitted for the fluctuation of it, meaning  $\gamma_s=\nu_\perp$ . This means that the exponents are in agreement with Fisher's scaling law [Eq. (12)] again. For the critical dynamical exponent  $Z=\nu_\parallel/\nu_\perp$  PC-like

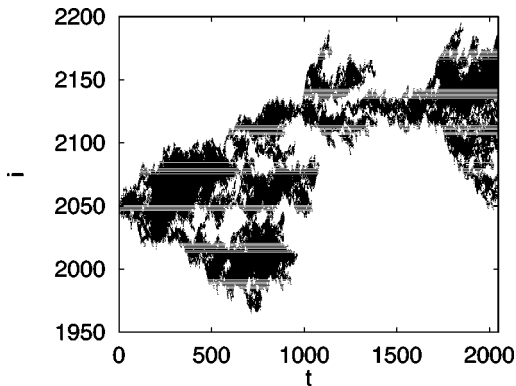


FIG. 18. Time evolution of the spin damage in the NEKIM model near the DS transition point.

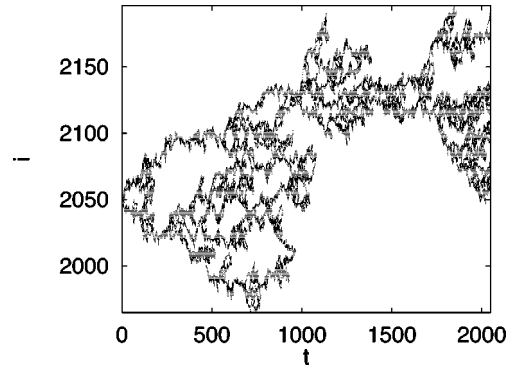


FIG. 19. Time evolution of the kink damage in the NEKIM model near the DS transition point.

scaling has been obtained for both the spin and kink damage cases. As Fig. 20 shows fitting can be done with  $L^{1.75}$  (PC value) in both cases. The deviation from DP scaling law can clearly be seen in the figure too. It is reasonable to assume that the scaling behaviors are inherited from the pure NEKIM model.

## VI. CONCLUSIONS

The damage spreading behavior of three one-dimensional, nonequilibrium models, exhibiting a parity conserving phase transition has been investigated numerically. The  $A$  SCA model of Grassberger has been found to be very sensitive on the initial conditions of the DS simulations. Acceptable results—which are in good agreement with that of the NEKIM model—can be found only when the DS simulations are started from steady state.

When the DS transition point coincided (within statistical error) with the ordinary critical point of the model (NEKIM,  $A$  model) interesting things have happened on both the spin and the kink damage level.

In the case of the  $B$  model the DS transition point was found to be far away from the critical point and all exponents (on spin and kink damage level equally) show DP universality class behavior independently of the parity conservation of damage variables.

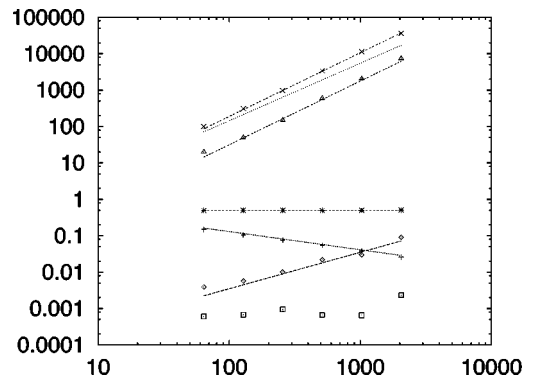


FIG. 20. Finite size scaling results for the NEKIM model. The crosses correspond to kink damage concentration, the squares to the fluctuations of it. The stars correspond to spin damage concentration, diamonds to the fluctuations of it. The triangles and the  $\times$ 's correspond to the kink and spin damage characteristic times ( $\tau$ ). Averaging was done over 500 surviving samples. The intermediate dotted line shows a DP-like scaling law for  $\tau$ .



TABLE II. Kink damage results.

	NEKIM	GR-A	GR-B	DP	PC
$p_c$	0.395(5)	0.1242(5)	0.539(1)		
$p_d$	0.395(5)	0.1242(5)	0.633(1)		
$\eta$	0.03(3)	0.03(3)	0.31(2)	0.3137(1)	0.0000(1)
$\delta$	0.28(1)	0.285(8)	0.160(2)	0.1596(4)	0.285(2)
$z$	1.14(1)	1.14(1)		1.2660(1)	1.141(2)
$Z$	1.74(3)	1.75(8)	1.59(4)	1.5798(2)	1.750(5)
$\beta/\nu_{\perp}$	0.500(6)	0.48(2)	0.27(2)	0.2522(6)	0.500(5)
$\gamma/\nu_{\perp}$	0.07(1)	0.1(1)	0.5	0.4956(2)	0.00(5)
Univ.	PC	PC	DP		

TABLE III. Spin damage results.

	NEKIM	GR-A	GR-B	DP	PC
$p_c$	0.395(5)	0.1242(5)	0.539(1)		
$p_d$	0.395(5)	0.1242(5)	0.633(1)		
$\eta'$	0.29(1)	0.29(2)	0.32(2)	0.3137(1)	0.285(2)
$\delta$	0.28(1)	0.285(8)	0.160(2)	0.1596(4)	0.285(2)
$z$	1.14(1)	1.14(1)		1.2660(1)	1.141(2)
$Z$	1.75(1)	1.79(5)	1.48(9)	1.5798(2)	1.750(5)
$\beta/\nu_{\perp}$	0.0001(1)	-0.001(1)	0.26(2)	0.2522(6)	0.500(5)
$\gamma/\nu_{\perp}$	1.00(7)	0.98(6)	0.46(4)	0.4956(2)	0.00(5)
Univ.	CPC	CPC	DP		

Tables II and III summarize the simulation results for the transition points and DS exponents of the models investigated as well as DP and PC critical exponent estimates from Refs. [18,27,37].

The  $\eta'$  is the PC exponent when the BAWe process is started from an odd number of particles. The CPC notation denotes the compact version PC universality class in analogy with the CDP in the case of DP universality. The conclusions of this paper are in agreement with previous works concerning PC to DP universality changes (for example, [24,27,31]). It is also in agreement with the recent DS study of Hinrichsen *et al.* [32], because when they obtained a PC class DS transition in the case of the one-dimensional kinetic Ising model they created a mixed dynamics, which satisfies both the BAWe parity conservation (i.e., they followed the kinks) and the symmetric ground state condition (i.e., they generated passive states by “switching” between two dynamics that results in double degeneration: no damage versus full damage). In our case it has never happened that one replica would become completely reflection symmetric to the other. The simple dynamical rules always have driven the states to the same sector, where spin and kink damages died out simultaneously.

One may assume this scheme to be valid for non-DS dynamical transitions too, if the model has multiple absorbing states (with or without  $Z_2$  symmetry). This hypothesis is

strengthened by simulations of the pure NEKIM model at the PC transition point. Starting the time dependent simulations from a single seed we measured the usual  $\eta$  and  $\delta$  exponents of the spins. In accordance with the DS results we obtained PC exponents again:  $\eta' = 0.285(5)$ ,  $\delta = 0.285(5)$ . A detailed study of CPC for the spins in the framework of NEKIM and its connection to the CDP point of Domany-Kinzel CA is under way [33].

These exponents satisfy the generalized hyperscaling law [13] with  $\beta = 0$ :

$$2 \left( 1 + \frac{\beta}{\beta'} \right) \delta' + 2 \eta' = dz, \quad (15)$$

where  $\beta'$  is the ultimate survival probability exponent. Note that the discontinuous phase transition, the compact clusters (see Fig. 18), and the form of the hyperscaling law suggests that our case is the parity conserving version of compact directed percolation [29,39,40].

#### ACKNOWLEDGMENTS

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